2020–2021学年秋冬学期

《数据结构基础》课程期末考试试卷

一、判断题（20分，共10题，每题2分）

1-1 The average run time and the extra space of Heapsort for sorting n elements are $O(n \log n)$ and $O(1)$, respectively.

1-2 The storage size of a graph using the adjacency matrix is only related to the number of vertices but has nothing to do with the number of edges.

1-3 For a connected graph, if there are exactly two vertices having odd degree, we can find an Euler tour that visits every vertex exactly once by starting from one of its odd-degree vertices.

1-4 In hashing, when the loading density approaches 1, the operation INSERTION will be seriously slowed down if the separate chaining method is used to solve collisions.

1-5 During sorting `(25，21，47，15，27，68，35，20)` in ascending order (升序) with the iterative version of Mergesort, `(15，21，25，47，27，68，35，20)` is the result of the second run.

1-6 In most restaurants, we follow one principle called "First come, first served". This principle can be implemented by a stack.

1-7 Consider two programs with time complexities being $T_1 = O(2n)$ and $T_2 = O(n \log n)$. then program 2 must run faster than program 1.

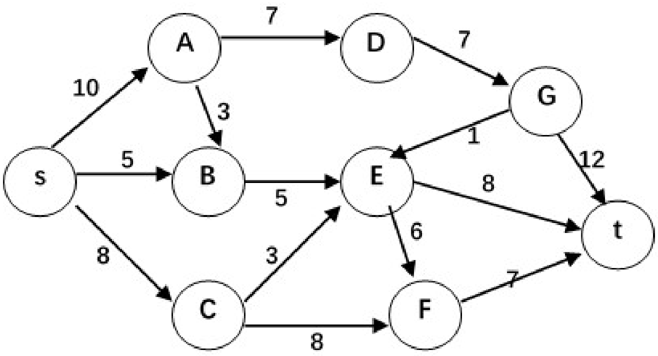
1-8 If a general tree `T` is converted into a binary tree `BT`, then the `BT`'s post-order traversal has the same sequence as that of the pre-order traversal of `T`.

1-9 For a binary tree, if its pre-order travel sequence is `{4，2，1，3，6，5，7}`, and its in-order travel sequence is `{1，2，3，4，5，6，7}`, then `4` is the parent of `3`.

1-10 The in-order traversal sequence of any min-heap must be in sorted order.

二、选择题（60分，共20题，每题3分）

2-1 The maximum flow in the network of the given Figure is:

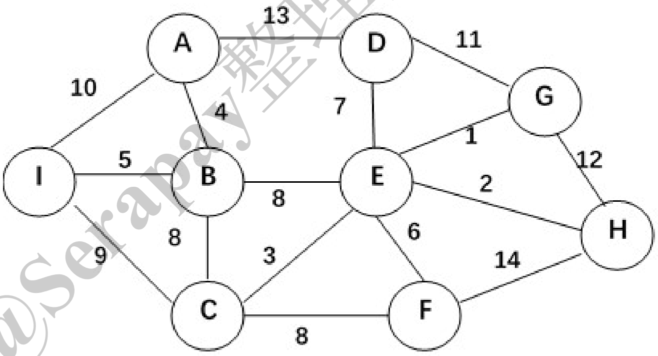


A. 21 B. 20 C. 22 D. 23

2-2 An inversion in an array `A[]` is any ordered pair `(i， j)` having the property that `i < j` but `A[i] > A[j]`. Given array `A`: `{3，87，12，61，70，26，45}`， after the first partition of Quicksort with Median3 pivot selection, the number of inversions will be decreased by ______.

A. 3 B. 9 C. 1 D. 6

2-3 Given a weighted graph as shown by the figure. Which one of the following statements is TRUE about its minimum spanning tree?



A. The minimum spanning tree is not unique and the total weight is 35.

B. If the weight of edge `(B， E)` was 7, then the minimum spanning tree would not be unique.

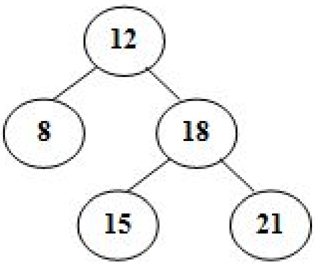
C. The minimum spanning tree is unique and the total weight is 36.

D. If the weight of edge `(B， C)` was 7, then the minimum spanning tree would be unique.

2-4 It is known that the fifth layer of a complete binary tree (the root is the first layer) has 8 leaf nodes, then the number of nodes of the complete binary tree is at most ______.

A. 52 B. 39 C. 31 D. 47

2-5 Which one of the following is **impossible** to be the insertion sequence of the given binary search tree?



A. `12， 8， 18， 21， 15` B. `12， 18， 15， 8， 21`

C. `12， 8， 15， 18， 21` D. `12， 18， 8， 21， 15`

2-6 Given `S` = `{1， 2， 3， 4， 5， 6， 7， 8， 9， 10， 11}` and 8 equivalence relations: `5`~`6`, `7`~`8`, `9`~`10`, `2`~`6`, `3`~`8`, `6`~`8`, `1`~`8`, `1`~`5`. After invoking successively these relations with union-by-size （if the two sizes are equal, the smaller element will be the root） and path compression, which one of the following statements is false?

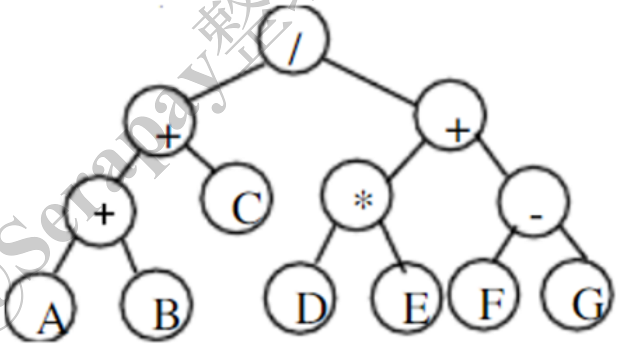
A. All elements are either a root or a child of a root.

B. Element `1` is a child of element `5`.

C. Element `2` and element `8` are siblings.

D. There are 4 equivalence classes.

2-7 The following binary tree is called an expression tree. Which one is the arithmetic expression that this tree represents?



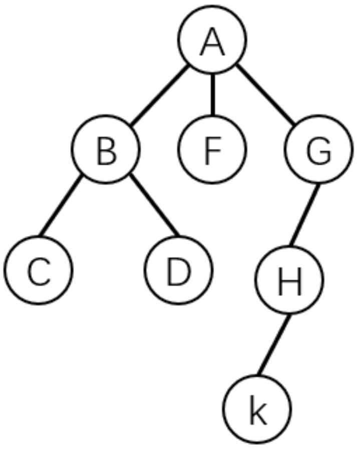
A. $(A+B+C)/(D*E)+(F-G)$

B. $(A+B+C)/(D*E+(F-G))$

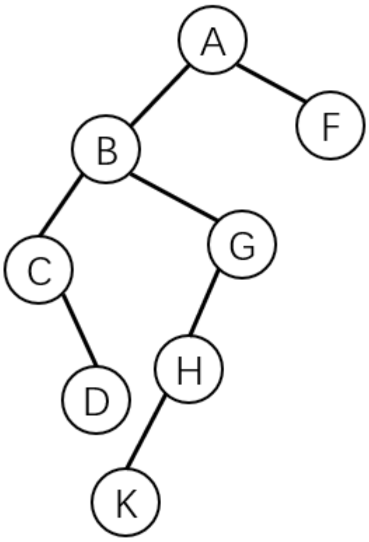
C. $A+B+C/(D*E)+(F-G)$

D. $A+B+C/D*E+F-G$

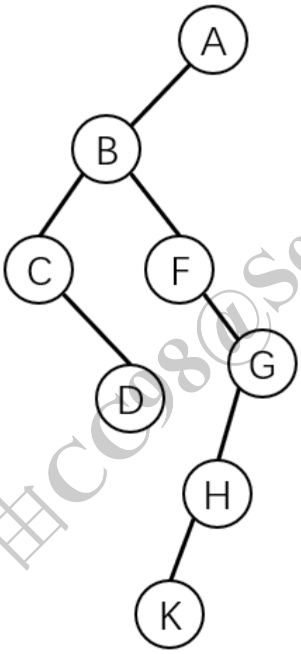
2-8 The following figure shows a tree. Which one is its corresponding binary tree with the "first child/next sibling" representation?

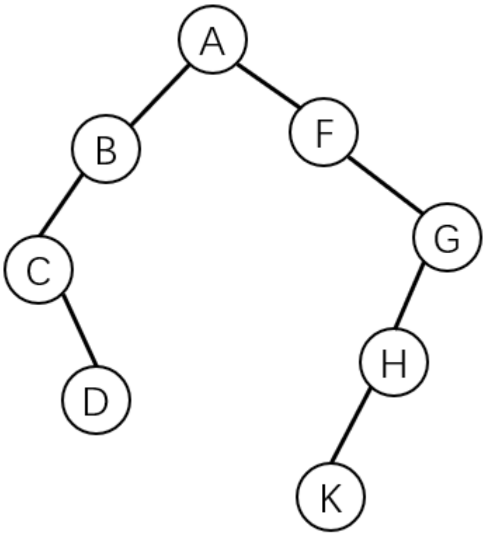


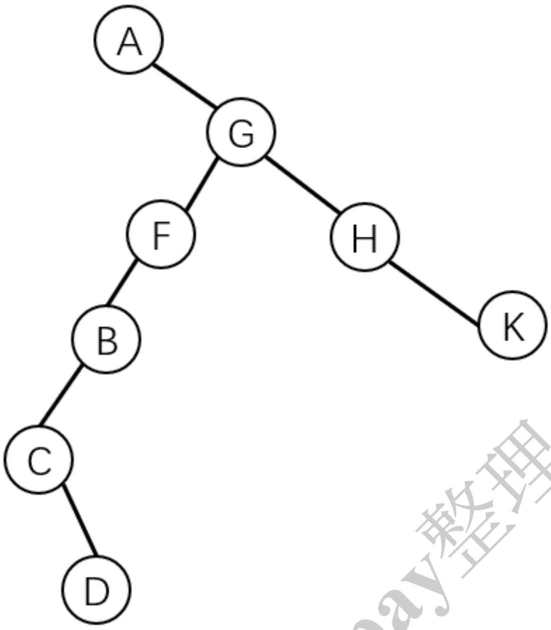
A.



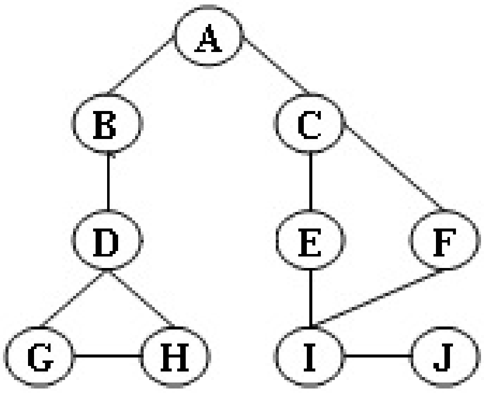
B.



C. 

D. 

2-9 The articulation points of the given graph are:



A. `A, B, I` B. `A, B, C, D, I` C. `A, B, C, D` D. `F, E, D`

2-10 In hashing with open addressing method，rehashing is definitely necessary when ______.

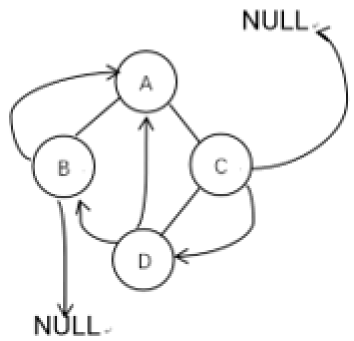
A. the hash table is half full

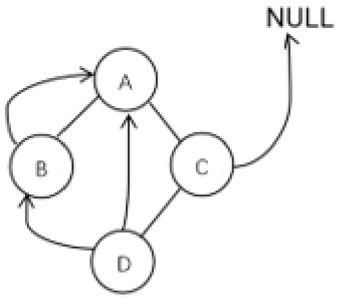
B. an insertion fails

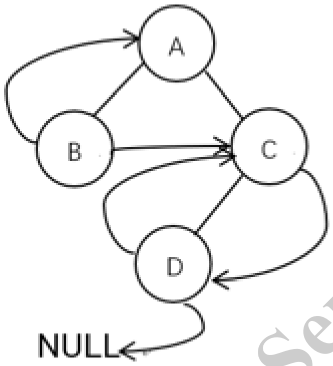
C. the hash function is not uniform

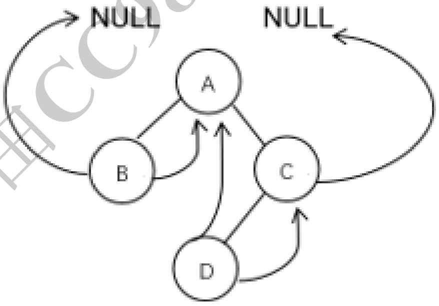
D. primary clustering occurs

2-11 Among the following threaded binary trees (the threads are represented by arrows), which one is the pre-order threaded tree?

A. 

B. 

C. 

D. 

2-12 Given an array of integers `{15, 22, 30, 18, 3, 8, 28}`. Build a min-heap using the linear algorithm and then call DeleteMin twice. Which of the following is the level-order traversal sequence of the remaining heap?

A. `15, 28, 18, 30, 22` B. `15, 28, 22, 18, 30`

C. `15, 18, 28, 22, 30` D. `15, 18, 28, 30, 22`

2-13 One of the following algorithms: Selection sort, Shell sort, Insertion sort, is applied to sort the sequence `(2， 12， 16， 88， 1， 5， 10)` in ascending order. If the resulting sequences of the first two runs are `(2， 12 ， 16， 10， 1， 5， 88)` and `(2， 12， 5， 10， 1， 16， 88)`, then the algorithm must be ______.

A. Uncertain B. Selection sort C. Insertion sort D. Shell sort

2-14 Given a binary tree, if its Pre-order traversal is `{A， B， C， D， E， G， F}` and the Post-order traversal is `{B， D， G， F， E， C， A}`, then which one is its In-order traversal?

A. `BADCGEF` B. `BGEAFCD` C. `BDACGEF` D. `BAGEFDC`

2-15 The following list is a series of operations for a stack:

- push 1
- push 2
- push 3
- pop
- pop
- push 4
- push 5
- pop
- push 6
- push 7
- pop
- pop
- pop
- push 8
- pop
- push 9
- pop
- pop

Which is the correct pop-up sequence?

A. `9 8 7 6 5 4 3 2 1` B. `3 2 5 7 6 4 8 9 1`

C. `1 2 3 4 5 6 7 8 9` D. `3 2 5 7 4 6 9 8 1`

2-16 Suppose `A` is an array of length N with some random numbers. What is the time complexity of the following program in the worst case?

```
void function( int A[], int N ) {
    int i, j = 0, cnt = 0;
    for (i = 0; i < N; ++i) {
        for (; j < N && A[j] <= A[i]; ++j);
        cnt += j - i;
    }
}
```

A. $O(N^{1.5})$ B. $O(N\log N)$ C. $O(N^2)$ D. $O(N)$

2-17 Let's traverse a complete binary tree in level-order, and define the level-order index of the root to be 1. Then for the i-th node visited, if its right child exists, then the index of this right child is _____.

A $2i$ B. $2i+1$ C. $2(i+1)+1$ D. $i+2$

2-18 Given the adjacency list of a directed graph,

```
a: e,f,g
b: Empty
c: Empty
d: b,c
e: Empty
f: d
g: c
```

which one below is NOT a valid topological order of the graph?

A. `a, e, f, d, b, g, c` B. `a, g, f, e, d, c, b`

C. `a, e, g, c, f, d, b` D. `a, f, e, d, g, b, c`

2-19 Given a hash table of size 13 (indexed from 0 to 12) with the hash function $H(Key) = Key\%11$, quadratic probing $H_i(key) = (H(key) + i^2)\%13$ is used to resolve collisions when the i-th($i > 0$) collision occurs. Then after inserting `{10, 21, 32, 33, 65, 12}` one by one into the hash table, which one of the following statements is false?

A. The loading density is less than 0.5.

B. The key `65` is at position `6`.

C. The key `12` is at position `1`.

D. The average search time is greater than `2`.

2-20 In a weighted undirected graph, if the length of the shortest path from `v1` to `v0` is 13, and there exists an edge of weight 2 between `v2` and `v1`, then which one of the following is correct?

A. The length of the shortest path from `v2` to `v0` must be no greater than 11.

B. The length of the shortest path from `v2` to `v0` must be greater than 15.

C. The length of the shortest path from `v2` to `v0` must be no less than 11.

D. The length of the shortest path from `v2` to `v0` must be less than 15.

三、程序填空题（12分，共4个空，每空3分）

5-1 Following function `Shellsort(int A[], int N)` is the implementation of Shellsort Algorithm with increment sequence `{1, 3, 7, 11}`. Please fill in the blanks.

```
void Shellsort(int A[ ], int N)
{
    int  i, j, k, Increment, Inc[] = {1, 3, 7, 11};
    int  Tmp;

    for (k = 3; k >= 0; k--)
    {
        __________;
        for (i = Increment; i < N; i++)
        {
            Tmp = A[i];
            for (j = i; j >= Increment; j -= Increment)
                if (Tmp < A[j - Increment])
                    __________;
                else
                    break;
            A[j] = Tmp;
        }
    }
}
```

5-2 The function `CountShortestPaths` is to find the number of shortest paths from `Vertex S` to every other vertices in a given graph (positive weights only). The distances and the numbers of shortest paths are stored in `dist[]` and `count[]`, respectively. The `Graph` is defined as follows:

```
typedef struct GNode *PtrToGNode;
struct GNode{
    int Nv;  /* Number of vertices */
    int Ne;  /* Number of edges    */
    int AdjMat[MaxVertexNum][MaxVertexNum];  /* adjacency matrix */
};
typedef PtrToGNode Graph;
```

Please fill in the blanks.

```
void CountShortestPaths(Graph G, Vertex S, bool known[], int dist[], int
count[])
{
    for (int i = 0; i < G->Nv; ++i)
    {
        known[i] = false;
        dist[i] = INFINITY;
        count[i] = 0;
    }

    dist[S] = 0;
    __________;

    while (true) {
        Vertex V = FindSmallestUnknown(G, known, dist);
        if (V == -1) break;
        known[V] = true;
        for (Vertex W = 0; W < G->Nv; ++W)
        {
            int weight = G->AdjMat[V][W];
            if (weight && !known[W])
            {
                if (dist[V] + weight < dist[W])
                {
                    dist[W] = dist[V] + weight;
                    count[W] = count[V];
                } else if (dist[V] + weight == dist[W])
                {
                    __________;
                }
            }
        }
    }
}
```

四、函数题（共8分）

6-1 Height of Binary Search Tree

You are supposed to write a function of finding the height of a binary search tree with the given post-order sequence.

Format of function:

```
int Height_of_BST( int postorder[], int N );
```

where the post-order sequence is stored in `int postorder[]`, and the integer `N` is the number of nodes in the tree, which is guaranteed to be positive. The function `Height_of_BST` is supposed to return the height of the binary search tree.

Note:

- It is guaranteed that the post-order sequence consists of distinct integers and does correspond to a binary search tree.
- You may assume that `MAXN` is a small number (less than 100) in the judge's program.

Sample program of judge:

```
#include <stdio.h>
#include <stdlib.h>

#define MAXN 10

int Height_of_BST( int postorder[], int N );

int main()
{
    int postorder[MAXN], N, i;
    scanf("%d", &N);
    for (i=0; i<N; i++) scanf("%d", &postorder[i]);
    printf("%d\n", Height_of_BST(postorder, N));

    return 0;
}

/* Your function will be put here */
```

Sample Input:

```
9
2 8 9 6 22 20 30 25 10
```

Sample Output:

```
3
```